

## PRIME NUMBERS THEORY

Grzegorz Hoppe

LET:

$$Q = N \setminus \left[ \bigcup_{n \in N} 2n \right] \setminus \left[ \bigcup_{n \in N} 3n \right] \setminus \left[ \bigcup_{n \in N} 5n \right] \setminus \left[ \bigcup_{n \in N} 7n \right]$$

BASED ON THE BASIC THEOREM OF ARITHMETIC:

$$\forall a \in N: \quad a = 2^n * 3^m * 5^s * 7^t * q$$

$$n, m, s, t \in N_{\{0\}}; \quad q \in Q;$$

THEN:

$$N = \left[ \bigcup_{n \in N_{\{0\}}} 2^n \right] \times \left[ \bigcup_{m \in N_{\{0\}}} 3^m \right] \times \left[ \bigcup_{s \in N_{\{0\}}} 5^s \right] \times \left[ \bigcup_{t \in N_{\{0\}}} 7^t \right] \times \left[ \bigcup_{q \in Q} q \right]$$

AND:

$$N = \left[ \bigcup_{n \in N_{\{0\}}} 2^n \right] \times \left[ \bigcup_{m \in N_{\{0\}}} 3^m \right] \times \left[ \bigcup_{s \in N_{\{0\}}} 5^s \right] \times \left[ \bigcup_{t \in N_{\{0\}}} 7^t \right] \times \left[ N \setminus \left[ \bigcup_{n \in N} 2n \right] \setminus \left[ \bigcup_{n \in N} 3n \right] \setminus \left[ \bigcup_{n \in N} 5n \right] \setminus \left[ \bigcup_{n \in N} 7n \right] \right]$$

AND THEN:

$$N = \left[ \bigcup_{n \in N} 2^n \right] \times \left[ \bigcup_{m \in N_{\{0\}}} 3^m \right] \times \left[ \bigcup_{s \in N_{\{0\}}} 5^s \right] \times \left[ \bigcup_{t \in N_{\{0\}}} 7^t \right] \times \left[ \bigcup_{q \in Q} q \right] \cup$$

$$\cup \left[ \bigcup_{m \in N} 3^m \right] \times \left[ \bigcup_{s \in N_{\{0\}}} 5^s \right] \times \left[ \bigcup_{t \in N_{\{0\}}} 7^t \right] \times \left[ \bigcup_{q \in Q} q \right] \cup$$

$$\cup \left[ \bigcup_{s \in N} 5^s \right] \times \left[ \bigcup_{t \in N_{\{0\}}} 7^t \right] \times \left[ \bigcup_{q \in Q} q \right] \cup$$

$$\cup \left[ \bigcup_{t \in N} 7^t \right] \times \left[ \bigcup_{q \in Q} q \right] \cup$$

$$\cup \left[ \bigcup_{q \in Q} q \right]$$

**CAUSE:**

$$\left[ \{1, 2\} \cup \left[ \bigcup_{n \in \mathbb{N}} 6n \right] \cup \left[ \bigcup_{n \in \mathbb{N}} (6n \pm 3) \right] \cup \left[ \bigcup_{n \in \mathbb{N}} (6n \pm 2) \right] \right] \supset \left[ \bigcup_{n \in \mathbb{N} \setminus \{0\}} 2^n \right] \times \left[ \bigcup_{m \in \mathbb{N} \setminus \{0\}} 3^m \right] \cap \left[ \bigcup_{n \in \mathbb{N}} (6n \pm 1) \right] = \emptyset$$

**THEN:**

$$\left[ \bigcup_{s \in \mathbb{N} \setminus \{0\}} 5^s \right] \times \left[ \bigcup_{t \in \mathbb{N} \setminus \{0\}} 7^t \right] \times \left[ \bigcup_{q \in Q} q \right] = \left[ \bigcup_{n \in \mathbb{N} \setminus \{0\}} (6n \pm 1) \right]$$

**LET:**

$$P_{1,11} = [P \cup \{1\}] \setminus \{2, 3, 5, 7\}$$

**THEN:**

$$Q = \left[ \{1\} \cup \left[ \bigcup_{q \in Q} q \right] \right] = \left[ \{1\} \cup \left[ \bigcup_{a \in \mathbb{N}; b \in P_{1,11}} b(6a \pm 1) \right] \right]$$

**AND:**

$$\left[ \bigcup_{s \in \mathbb{N} \setminus \{0\}} 5^s \right] \times \left[ \bigcup_{t \in \mathbb{N} \setminus \{0\}} 7^t \right] \times \left[ \bigcup_{q \in Q} q \right] = \left[ \bigcup_{n \in \mathbb{N} \setminus \{0\}; b \in \{5, 7\}} b(6n \pm 1) \right]$$

**AND THEN:**

**for b = 1**

$$P \subset \left[ \{2, 3, 5, 7\} \cup \left[ \bigcup_{a > 1 \in \mathbb{N}} (6a \pm 1) \right] \right]$$

elimination of the numbers  $(6a \pm 1)$  in the second and next powers

**CAUSE:**

$$\forall a \in \mathbb{N}$$

$$(6a + 1) * (6a + 1) = 6a(6a + 2) + 1$$

$$(6a - 1) * (6a - 1) = 6a(6a - 2) + 1$$

$$(6a - 1) * (6a + 1) = 6a^2 - 1$$

**THEN:**

$$(6a + 1) \neq (6n(6n + 2) + 1) \rightarrow a \neq n(6n + 2) \rightarrow \text{if } a \neq 6n + 2; n \in \mathbb{N}$$

$$(6a - 1) \neq (6n(6n - 2) + 1) \rightarrow 6a \neq 6n(6n - 2) + 2 \rightarrow \text{if } a \in \mathbb{N}$$

$$(6a - 1) \neq 6a^2 - 1 \rightarrow 6a \neq 6a^2 \rightarrow \text{if } a \neq a^2 \rightarrow \text{if } a \neq 1$$

elimination of the numbers divisible by 5 or 7 from all other numbers in form  $(6a \pm 1)$

**CAUSE:**

$$\forall n, m \in \mathbb{N}$$

$$(6n \pm 1) \neq 5(6m \pm 1) \rightarrow 6n \neq 30m \pm 5 \pm 1$$

$$(30m \pm 5 \pm 1) \neq 6n \rightarrow n \neq 5m \pm (5 \pm 1) * \frac{1}{6}$$

**THEN:**

$$\rightarrow [n \neq (5m + 1); \text{ for numbers } (6n - 1)]$$

$$\rightarrow [\text{and } n \neq (5m - 1); \text{ for numbers } (6n + 1)]$$

**CAUSE:**

$$\forall n, m \in \mathbb{N}$$

$$(6n \pm 1) \neq 7(6m \pm 1) \rightarrow 6n \neq 42m \pm 7 \pm 1$$

$$(42m \pm 7 \pm 1) \neq 6n \rightarrow n \neq 7m \pm (7 \pm 1) * \frac{1}{6}$$

**THEN:**

$$\rightarrow [n \neq (7m + 1); \text{ for numbers } (6n + 1)]$$

$$\rightarrow [\text{and } n \neq (7m - 1); \text{ for numbers } (6n - 1)]$$

**THEN:**

$$\begin{aligned} P &= \{2, 3, 5, 7\} \cup \\ &\cup \{p|p = (2^{n+1} * 3^{m+1} * q - 1); n, m \in N; q \in Q\} \setminus \{35\} \\ &\cup \{p|p = (2 * 3^{n+1} * q - 1); n \in N; q \in Q\} \setminus \{35\} \\ &\cup \{p|p = (3 * 2^{n+1} * q - 1); n \in N; q \in Q\} \setminus \{35\} \\ &\cup \{p|p = (2^{n+1} * 3^{m+1} * q + 1); n, m \in N; q \in Q\} \\ &\cup \{p|p = (3 * 2^{2n+2} * q + 1); n \in N; q \in Q\} \\ &\cup \{p|p = (12 * q + 1); q \in Q\} \\ &\cup \{p|p = (30 * q + 1); q \in [Q \setminus P]\} \\ &\cup \{p|p = (42 * q + 1); q \in [Q \setminus P]\} \end{aligned}$$

**AND THEN:**

**Set of Prime Numbers P:**

**is a sum of sets:  $\{2, 3, 5, 7\}$  and set of all  $p|p = (6a \pm 1)$ ;**

**such that:**

$$p = (6a - 1);$$

$$\forall a \neq 6; a \in \{\{5, 7\} \cup [6N \setminus 5N \setminus 7N] \cup [3N \setminus 4N \setminus 5N \setminus 7N] \cup [2N \setminus 5N \setminus 7N \setminus 9N]\}$$

$$p = (6a + 1);$$

$$\forall a \in \{[6N \setminus 5N \setminus 7N] \cup [12N \setminus 5N \setminus 7N \setminus 9N \setminus 24N]\}$$

$$p = (6 * 2q + 1); \forall q \in Q;$$

$$p = (6 * 5q + 1); \forall q \in [Q \setminus P]$$

$$p = (6 * 7q + 1); \forall q \in [Q \setminus P]$$

**Where:**

$$Q = N \setminus \left[ \bigcup_{n \in N} 2n \right] \setminus \left[ \bigcup_{n \in N} 3n \right] \setminus \left[ \bigcup_{n \in N} 5n \right] \setminus \left[ \bigcup_{n \in N} 7n \right]$$