The Proof of Goldbach Hypothesis

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The presented Numbers Theory is based on the basic theorem of arithmetic, that any number that is not a prime number or number 1, can be represented by the product of prime numbers. The most important assumption of the theory is the division of all natural numbers into the original and separable subsets, according to the divisibility of numbers. The important assumption is to divide numbers into following separate subsets:

a) numbers divisible by number 2,
b) numbers divisible by number 3 and at the same time indivisible by number 2,
c) numbers divisible by number 5 and at the same time indivisible by numbers 2 or 3,
d) numbers divisible by number 7 and at the same time indivisible by numbers 2, 3 or 5,
e) and numbers indivisible by any of the numbers 2,3,5 or 7.

Assumptions:

N - set of natural numbers
P - set of prime numbers

Definitions:

Definition 1.1.

$P_{1,11} = (P \cup \{1\}) \setminus \{2, 3, 5, 7\}$,
$P_{1,11} = \{1, 11, 13, 17, 19, \ldots\}$

Definition 1.2.

Q $-$ set of all numbers that have at least two prime factors, none of which are 2,3,5 or 7, and the number 1.
$Q = \{1, 121, 143, 169, \ldots\}$

Lemma 1.3. Based on basic theorem of arthmetic, that any number that is not a prime number or number 1, can be represented by the product of prime numbers:

then

$$\forall_{n>1} \in N: \; n = 2^n \cdot 3^m \cdot 5^s \cdot 7^t \cdot p \cdot q; \; n, m, s, t \in (N \cup \{0\}); \; p \in P_{1,11}; \; q \in Q$$

and then:

$$N = \{2^{N\{0\}}\} \times \{3^{N\{0\}}\} \times \{5^{N\{0\}}\} \times \{7^{N\{0\}}\} \times P_{1,11} \times Q$$

Lemma 1.4. Because:

$$\forall_n \in N: \; n = (6a + \{1,2,3,4,5,6\}) \text{ or } n \in \{1,2,3,4,5,6\}; \; a \in N$$

then:

$$\begin{aligned} N =& \{1,2,3,4,5\} \cup 6N \cup (12N \pm \{1,2,3,4,5\}) = (12N + \{0,1,2,3,4,5,6\}) \cup \\ & \cup (12N \setminus \{1,2,3,4,5\}) \cup \{1,2,3,4,5,6\} \end{aligned}$$

Assumption 1.5. $N_2 = 2N \setminus \{2\}$

Ordering of set of natural numbers:

Based on (1.4),
and because:

$12N = 6 \times 2N = 6(N_2 \cup \{2\})$

Then:

$$\begin{aligned} N &= (6(N_2 \cup \{2\}) + \{0,1,2,3,4,5,6\}) \cup (6(N_2 \cup \{2\}) - \{1,2,3,4,5\}) \cup \{1,2,3,4,5,6\} \\ N &= (6N_2 + \{0,1,2,3,4,5,6\}) \cup (6N_2 - \{1,2,3,4,5\}) \cup \{1,\ldots,18\} \\ N &= [(6N_2 + \{0,2,4,6\}) \cup (6N_2 - \{2,4\})] \cup (6N_2 \pm \{3\}) \cup (6N_2 \pm \{1,5\}) \cup \{1,\ldots,18\} \end{aligned} \tag{1.6}$$

Let:

$$N_3 = ((6N_2 \pm \{3\}) \cup \{9,15\}) = \{3\} \times \{3^{N \cup \{0\}}\} \times \{5^{N \cup \{0\}}\} \times \{7^{N \cup \{0\}}\} \times P_{1,11} \times Q \tag{1.7}$$

$$\forall_n \in N_3: \; (n \bmod 12) = (3,9)$$

Then:

$$N = N_2 \cup N_3 \cup (6N_2 \pm \{1,5\}) \cup \{1,\ldots,18\} \tag{1.8}$$

Let:

$$N_5 = (30N \pm \{5\}) = \{5\} \times \{5^{N \cup \{0\}}\} \times \{7^{N \cup \{0\}}\} \times P_{1,11} \times Q \tag{1.9}$$
$$\forall_n \in N_5 : \ (n \bmod 30) = (5, 25)$$

$$N_7 = ((42N \pm \{7\}) \setminus \{35\}) = \{7\} \times \{7^{N \cup \{0\}}\} \times P_{1,11} \times Q \tag{1.10}$$
$$\forall_n \in N_7 : \ (n \bmod 42) = (7, 35)$$

Because (1.8):

$$N = \{2, 3, 5, 7\} \cup N_2 \cup N_3 \cup N_5 \cup P_{1,11} \cup Q \tag{1.11}$$

Then:

$$(6N_2 \pm \{1, 5\}) \cup \{1\} = N_5 \cup N_7 \cup P_{1,11} \cup Q \tag{1.12}$$

Ordering of set $P_{1,11}$ and set Q:

Based on (1.9),(1.10),(1.12):

$$[P_{1,11} \cup Q] = [(6N_2 \pm \{1, 5\}) \cup \{1\}] \setminus [(30N \pm \{5\}) \cup (42N \pm \{7\})] \tag{1.13}$$
$$\forall_n \in [(P_{1,11} \cup Q) \setminus \{1\}] : \ (n \bmod 12) = (1, 5, 7, 11)$$

Sum of numbers from sets: $P_{1,11}, Q$ and $\{3, 5, 7\}$:

Based on (1.13):

$$\begin{aligned} &[(P_{1,11} \cup Q) + \{3, 5, 7\}] = [(6N_2 + \{-2, 0, 2, 4, 6, 8, 10, 12\}) \cup \{4, 6, 8\}] \setminus \\ &\setminus ([(30N + \{-5, 5\}) \cup (42N + \{-7, 7\})] + \{3, 5, 7\}) = \\ &= N_2 \setminus [(30N + \{-2, 0, 2, 8, 10, 12\}) \cup (42N + \{-4, -2, 0, 10, 12, 14\})] \end{aligned} \tag{1.14}$$

Based on (1.13):

$$\begin{aligned} &[(P_{1,11} \cup Q) + (P_{1,11} \cup Q)] = \\ &= [((6N_2 \pm \{1, 5\}) \cup \{1\}) + (6N2 \pm \{1, 5\}) \cup \{1\}] \setminus \\ &\setminus ([(30N \pm \{5\}) \cup (42N \pm \{7\})] + [(30N \pm \{5\}) \cup (42N \pm \{7\})]) = \\ &= ((6N_2 + \{-4, -2, 2, 4, 6, 10\}) \cup \{2\}) \setminus \\ &\setminus [2(30N \pm \{5\}) \cup 2(42N \pm \{7\}) \cup ((30N \pm \{5\}) + (42N \pm \{7\}))] = \\ &= N_2 \setminus [2(30N \pm \{5\}) \cup 2(42N \pm \{7\}) \cup ((30N \pm \{5\}) + (42N \pm \{7\}))] \end{aligned} \tag{1.15}$$

Because:

$$\begin{aligned}
&[(30N + \{-2, 0, 2, 8, 10, 12\}) \cup (42N + \{-4, -2, 0, 10, 12, 14\})] \cap \\
&\cap [2(30N \pm \{5\}) \cup 2(42N \pm \{7\}) \cup ((30N \pm \{5\}) + (42N \pm \{7\}))] = \emptyset
\end{aligned} \tag{1.16}$$

Then:

$$\begin{aligned}
&[(P_{1,11} \cup Q) + \{3, 5, 7\}] \cup [(P_{1,11} \cup Q) + (P_{1,11} \cup Q)] = \\
&= N_2 \cup \{2\} \setminus \{10, 12\}
\end{aligned} \tag{1.17}$$

Sum of two prime numbers $\boldsymbol{p}$

Because:

$$(P_{1,11} \cap Q) = \{1\} \tag{1.18}$$

Then:

$$\begin{aligned}
&[P_{1,11} + Q] \cap (Q + Q) = \{2\} \\
&[P_{1,11} + P_{1,11}] \cap (Q + Q) = \{2\} \\
&(P_{1,11} + \{2\}) \cap (Q + \{2\}) = \{3\} \\
&[Q + \{3, 5, 7\}] \cap (Q + Q) = \emptyset
\end{aligned} \tag{1.19}$$

And then:

$$\begin{aligned}
&[(Q + Q) \cup (Q + \{3, 5, 7\})] \subset [(P_{1,11} + P_{1,11}) \cup (P_{1,11} + \{3, 5, 7\})] \\
&[(P_{1,11} + \{3, 5, 7\}) \cup (P_{1,11} + P_{1,11})] = N_2 \setminus \{10, 12\}
\end{aligned} \tag{1.20}$$

Based on (1.1),(1.20):

$$\begin{aligned}
&P = \{2\} \cup \{3, 5, 7\} \cup P_{1,11} \setminus \{1\} \\
&(P + P) = (\{2\} \cup \{3, 5, 7\} \cup (P_{1,11} \setminus \{1\})) + (\{2\} \cup \{3, 5, 7\} \cup (P_{1,11} \setminus \{1\})) = \\
&= \{4\} \cup \{6, 8, 10, 12, 14\} \cup \{5, 7, 9\} \cup [(P_{1,11} + \{3, 5, 7\}) \cup (P_{1,11} + P_{1,11})] \cup (P_{1,11} + \{2\}) = \\
&= \{4, 5, 6, 7, 8, 9, 10, 12, 14\} \cup (N_2 \setminus \{10, 12\}) \cup (P_{1,11} + \{2\}) = \\
&= (N_2 \cup (P_{1,11} + \{2\}) \cup \{5, 7, 9\}) \setminus \{3\}
\end{aligned}$$

And then:

$$\boldsymbol{N_2 \subset (P + P)} \tag{1.21}$$

based on (1.9),(1.10),(1.13):

$$(P_{1,11} \cup Q) = [(12N \pm \{1\}) \cup (12 \pm \{5\})] \setminus [(30N \pm \{5\}) \cup (42N \pm \{7\})] \tag{1.22}$$

and

$$(P_{1,11} \cup Q) \bmod 24 = (1, 5, 7, 11, 13, 17, 19, 23) \tag{1.23}$$

based on (1.23)

$$\text{for } p > 23: \;\; P \bmod 24 = (1, 5, 7, 11, 13, 17, 19, 23) \tag{1.24}$$

based on (1.22)

$$Q \bmod 24 = (1, 5, 11, 13, 19, 23) \tag{1.25}$$

and

$$\begin{aligned} Q \bmod 144 = (&1, 5, 11, 13, 23, 25, 35, 37, 47, 49, 53, 59, 61, 67, 71, 73, \\ &77, 83, 85, 91, 95, 97, 107, 109, 119, 121, 133, 139, 143) \end{aligned} \tag{1.26}$$

based on (1.26)

$$\text{for } p > 139 \;\; P \bmod 144 = (7, 17, 19, 29, 31, 41, 43, 79, 89, 101, 103, 111, 113, 127, 131, 137) \tag{1.27}$$

Theorem 1.
Based on (1.1-1.27)

$$\forall_n > 23 \in \mathbb{N} \tag{1.28}$$

if: $n \bmod 24 = (7, 17)$, and if n is not divisible by 3,5 or 7, then $n \in P$
if: $n \bmod 24 = (1, 5, 11, 13, 19, 23)$,
and: $n \bmod 144 = (7, 17, 19, 29, 31, 41, 43, 79, 89, 101, 103, 111, 113, 127, 131, 137)$
and if n is not divisible by 3,5 or 7, then $n \in P$

Summary:
All natural numbers are ordered in following subsets:

$$N = \{1, 2, 3, 5, 7\} \cup N_2 \cup N_3 \cup N_5 \cup N_7 \cup (P_{1,11} \setminus \{1\}) \cup (Q \setminus \{1\})$$

where:

$\forall_n \in N_2 : \ (n \bmod 12) = (0, 2, 4, 6, 8, 10)$

$\forall_n \in N_3 : \ (n \bmod 12) = (3, 9)$

$\forall_n \in N_5 : \ (n \bmod 30) = (5, 25)$

$\forall_n \in N_7 : \ (n \bmod 42) = (7, 35)$

$\forall_{n>1} \in (P_{1,11} \cup Q) : \ (n \bmod 12) = (1, 5, 7, 11)$

$\forall_{p>1} \in P_{1,11} : \ (p \bmod 24) = (1, 5, 7, 11, 13, 17, 19, 23)$

$\forall_{n>1} \in Q : \ (n \bmod 24) = (1, 5, 11, 13, 19, 23)$

$\forall_{p>144} \in P : \ (p \bmod 144) = (7, 17, 19, 29, 31, 41, 43, 79, 89, 101, 103, 111, 113, 127, 131, 137)$

$$\forall_{n>1} \in Q : \ (n \bmod 144) = (1, 5, 11, 13, 23, 25, 35, 37, 47, 49, 53, 59, 61, 67, 71, 73, 77, 83, 85, 91, 95, 97, 107, 109, 119, 121, 133, 139, 143)$$